Enrollment No: ______________________ **Exam Seat No:**______________________

C.U.SHAH UNIVERSITY

Winter Examination-2022

Subject Name: Engineering Mathematics-I

Subject Code: 4TE01EMT3 **Branch: B.Tech (All)**

Semester: 1 **Date: 09/01/2023** **Time: 11:00 To 02:00** **Marks: 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) The principal argument of $z = 1 + i$. (01)

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{5}$ (b) $\frac{\pi}{6}$

(b) The conjugate of $z = \frac{(5-3i)}{4+3i}$ is _____. (01)

(a) $\frac{11}{5} + \frac{27}{5}i$ (b) $\frac{11}{5} - \frac{27}{5}i$ (c) $\frac{27}{5} - \frac{11}{5}i$ (d) $\frac{27}{5} + \frac{11}{5}i$

(c) The Principal value of $\log(2+2i)$ is ? (01)

(a) $\log 2 + i\frac{\pi}{4}$ (b) $\log 2 - i\frac{\pi}{4}$ (c) $-\frac{1}{2}\log 2 + i\frac{\pi}{4}$ (d) $\frac{1}{2}\log 2 + i\frac{\pi}{4}$

(d) The of Eigen values of matrix $A = \begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix}$ are _____. (01)

(a) 1,3 (b) 2,3 (c) 1,4 (d) 2,4

(e) If $y = \sin(ax+b)$ then $y_n =$ ________. (01)

(a) $b^2 \sin(ax + b + n\frac{\pi}{2})$ (b) $a^2 \cos(ax + b + n\frac{\pi}{2})$

(c) $a^n \sin(ax + b + n\frac{\pi}{2})$ (d) $a^n \cos(ax + b + n\frac{\pi}{2})$

(f) If $y = e^{3x}$ then $y_n =$ ________. (01)

(a) $3^{n+1}e^{3x}$ (b)$3^{n-1}e^{3x}$ (c)$3^n e^{3x}$ (d) $3^{n+1}e^{4x}$

(g) The Maclaurin's series of $\cos x$ is _______. (01)

(a)$x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}$ (b) $1 + \frac{x^2}{2!} - \frac{x^4}{4!} - \frac{x^6}{6!}$ (c) $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$ (d) $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}$

(h) Find $\lim_{x\to\frac{\pi}{2}} \sin x$. (01)

(a) 0 (b) 1 (c) -1 (d) 2

(i) Find $\lim_{x\to 1} \frac{x^2-2x+1}{2x^2-5x+4}$. (01)

(a) 0 (b) $-\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $-\frac{1}{3}$

(j) The value of $\lim_{\substack{x\to 1 \\ y\to 3}} \frac{x^3+y^3}{x^2+y^2}$ is______. (01)

(a) $\frac{14}{5}$ (b) $\frac{5}{14}$ (c) 5 (d) 14

(k) The degree of Homogeneous equation $f(x,y) = x^3 + x^2y + y^3$ is_____. (01)

(a) 2 (b) 3 (c) -2 (d) -3

(l) The value of $\lim_{x\to 0} \frac{3^x - 2^x}{x}$ is_____. (01)

(a) $\log\frac{2}{3}$ (b) $\log\frac{3}{2}$ (c) $\log\frac{9}{4}$ (d) $\log\frac{4}{9}$

(m) For which value k vectors $v_1 = (1,k), v_2 = (2k,2)$, are linearly dependent. (01)

(a) $k = 4$ (b) $k = 3$ (c) $k = 1$ (d) $k = 5$

(n) The rank of matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 4 \\ 1 & 4 & 5 \end{bmatrix}$ is______. (01)

(a) 3 (b) 2 (c) 1 (d) 0

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

(a) If $y = e^\wedge(a\sin^{-1} x)$ $x \in (-1,1)$ then prove that $(1-x^2)y_2 - xy_1 - a^2y = 0$. (05)

(b) If $y = \cos(ax+b)$ a, b are constant with $a \neq 0$ then prove that (05)

$y_n = \left\{\cos\left(ax + b + n\left(\frac{\pi}{2}\right)\right)\right\}$ $n \in N$.

(c) If $y = \frac{1}{(2x+1)(3x+1)}$ $, x \neq -\frac{1}{3}, -\frac{1}{2}$ then find y_n. (04)

Q-3 **Attempt all questions** **(14)**

(a) Find a Maclaurin's series for $f(x) = \cos x$. (05)

(b) Expand f$(x) = \log x$ in power of $(x-2)$. (05)

(c) Expand $f(x) = x^4 - 11x^3 + 43x^2 - 6x + 14$ acsending power of $(x-1)$. (04)

Q-4 **Attempt all questions** **(14)**

(a) Find $\lim_{x\to y} \frac{x^y - y^x}{x^x - y^y}$, where y is constant. (06)

(b) Find $\lim_{x\to 0} \frac{tanx - x}{x^3}$. (04)

(c) Evaluate $\lim_{x\to 0} \frac{a^x + b^x + c^x}{3}$. (04)

Q-5 **Attempt all questions** **(14)**

(a) Find the roots of the equation $z^5 = 1$. (05)

(b) Prove that (05)

$(1 + cos\theta + i\ sin\theta)^n + (1 + cos\theta - isin\theta)^n = 2^{n+1}\cos^n\frac{\theta}{2}\cos\left(\frac{n\theta}{2}\right)$.

(c) Express $\frac{(1+i)(1-i)}{(4-3i)(4+3i)}$ in the form of $a + ib$. (04)

Q-6 **Attempt all questions** **(14)**

(a) Verify Caley-Hamilton theorem for matrix $A = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ (06)

(b) Find the Eigen Values of the matrix $A = \begin{bmatrix} 1 & 0 & 3 \\ 1 & 4 & 5 \\ 1 & 4 & 6 \end{bmatrix}$ (05)

	(c)	Find the rank of matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$	(03)
Q-7		**Attempt all questions**	**(14)**
	(a)	Find the value of $\lim\limits_{\substack{x \to 0 \\ y \to 0}} \dfrac{x^3 - y^3}{x^2 + y^2}, \quad x \neq 0, y \neq 0$	(05)
	(b)	If $u = e^{xyz}$ then prove that $\dfrac{\partial^3 u}{\partial x \partial y \partial z} = (1 + 3xyz + x^2y^2z^2)e^{xyz}$.	(05)
	(c)	If $u = tan^{-1}\dfrac{y}{x}$ then find $\dfrac{\partial^2 u}{\partial x \partial y}$.	(04)
Q-8		**Attempt all questions**	**(14)**
	(a)	Express $\cos 8\theta$ in terms of $\cos\theta$ and $\sin\theta$.	(06)
	(b)	Solve system of linear equations by using Gauss-elimination method $3x + y - z = 3\ , x - 2y + 9z = 8\ , 2x - 8y + z = -5$.	(05)
	(c)	Evaluate $\lim\limits_{x \to 0} \dfrac{e^x + e^{-x} - x^2 - 2}{sin^2 x - x^2}$.	(03)